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THEORETICAL JUSTIFICATION OF DETERMINING THE AREAS OF LAND FOR THE PURPOSES OF 3D CADASTRE

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Abstract

When switching to 3D cadastre in the Russian Federation for cadastral registration of both capital construction projects and land plots, certain accounting methods and amendments to existing laws and regulations will be needed. For registration in such a real estate register of a land plot, it is advisable to enter information about both the area of the projection of the land plot on the plane and the actual area of the land plot. Previously, various methods of calculating the actual area of the land plot were described: by dividing the plot into triangles and summing the areas of inclined equilateral or arbitrary triangles; by including corrections for the slope of the terrain in the formula for calculating the area; through splines and so on. Two methods have been developed, using which it will be possible to calculate the actual area of land plots with sufficient accuracy, spending less time.

1 Introduction

In the Russian Federation, real estate registration in the Unified State Register of Real Estate is conducted in a two-dimensional form. It means that the coordinates of the specific points of the land plot (land plot part) boundaries, the coordinates of the specific points of the building contour, structure, parts of such real estate property, the coordinates of the specific points of the contour lines of construction-in-progress are entered in the Unified State Register of Real Estate register in two-dimensional form.

In order to record volumetric objects, the attempts were made to introduce a three-dimensional spatial description of real estate property (3D modeling) in the Unified State Register of Real Estate. This approach was primarily aimed at solving the problems arisen at the entry in the state register of real estate for state cadastral registration of real property located at different levels (underground, ground and above-ground levels), including buildings of complex configuration, bridges, utility infrastructure, underground structures and other objects associated with the mapping of structural element projections of the said property items.

The same is applied to land plots, because the actual area of a land plot and the area of a plot projection onto the plane, which is currently documented and recorded in the Unified State Register of Real Estate, are different.

There is a legislative possibility of recording three-dimensional models of real property object (3D-models) in the Unified State Register of Real Estate, availability of geoinformation technologies, up-to-date geodetic equipment and software suitable for a three-dimensional cadastre. But in fact, there is no 3D real estate cadastre in the Russian Federation.

This is mainly due to the fact that there is no appropriate scientific and methodological base for implementation of cadastral work for a three-dimensional cadastre of real property at the time of land plot registration, taking into account the land topography: methods of differentiating real property objects according to certain characteristics, methods and technology for systematizing three-dimensional

data, degree of detail and in-depth survey of land topography and the required accuracy of determining the turning point coordinates of the land plot boundaries and topographic positions.

Currently, there are several methods for calculating the actual area of land plots. A calculating method for the areas of two polygons with a common side is implemented and described in the article by V. A. Katushkova, V. M. Serdyukov [1]. In order to reduce the error associated with distortions of projection onto a plane, A. V. Vinogradov takes the reference ellipsoid as a single reference surface. This method eliminates the need of coordinates recalculation when shifting from one zone to another zone [2]. This method also suggests the determination of the physical surface of the site area using the iteration method [3]. W. D. Samratov suggested the application of a topocentric horizontal coordinate system, where the axis of the applicate system is aligned at each point with the direction of the normal to the earth's ellipsoid surface, and the main coordinate plane is aligned with the plane of the local horizon [4]. K. S. Clark suggests using the spatial equivalent of the method of "erratic motion of magnetic pole on the plane": taking the corners of the polar grid squares from the central topographic model, calculating their mean in each square, dividing the square into four triangles with a common vertex at a point with the middle elevation, and calculating the surface area of the prism, formed by the heights of the triangle vertices [5]. V. A. Byvshev considers the calculation of the orthogonal projection site area onto the reference ellipsoid and the area of the Gauss-Kruger projection from the reference ellipsoid to the plane. He developed an algorithm for determining the area section of the Earth's surface using topographic and geodetic information. In this method, justification differential splines and integral and differential calculus are used [6]. A. I. Kalenitsky and E. E. Vasilyeva substantiated an informative indicator characterizing the degree of topography dissection. Their application in calculations allows the development of the methodology and justifying a high-performance automated computer technology for express

assessment of the actual expected size of the physical surface area of sites and areas [7]. A.V. Nikitin proposed to determine the land plots area using a spatial geometric model of the area [8,9]. G. G. Astashenkov, G. E. Strelnikov, and V.Y. Shipulin described the method for determining the actual value of the slope terrain area using field measurements [10].

Although, to cover a large area, less costly and quickly implemented methods for determining the actual areas of land plots will be sought after. They can be used to calculate areas, record and organize them in a three-dimensional real estate cadastre.

2 Methods and materials

Determining of areas and entering data into the Unified State Register of Real Estate on the actual area of land plots for a three-dimensional real estate cadastre should be mostly applied for providing interested parties with the information about the actual area of land plots, and not for the purpose of collecting rental payments and taxes. For example, information on

the actual area will be relevant for the technical and biological recultivation of a worked-out open pit with flattened sides, since the area of the open pit surface increases after development. Or, for example, information on the actual area of a highway with slopes will be useful for servicing the highway, considering the roadbed and slopes.

Currently, special attention is paid to obtain data on the actual position of real property objects (the coordinates of their boundary turning points) for the Unified State Register of Real Estate by a photogrammetric method. It also describes the accuracy of determining the coordinates of specific points by the photogrammetric method using aerial photography and satellite imagery for the relevant land category and permitted use of land plots.

The photogrammetric method of data acquisition speeds up the information collection about the turning point coordinates of real estate boundaries in three-dimensional space and allows precise calculations of the earth's surface area using certain techniques, since the distance between the points obtained is quite small. Figures 1 and 2 show data on the distance between topography points obtained by the photogrammetric method using the CREDO 3D SCAN program.

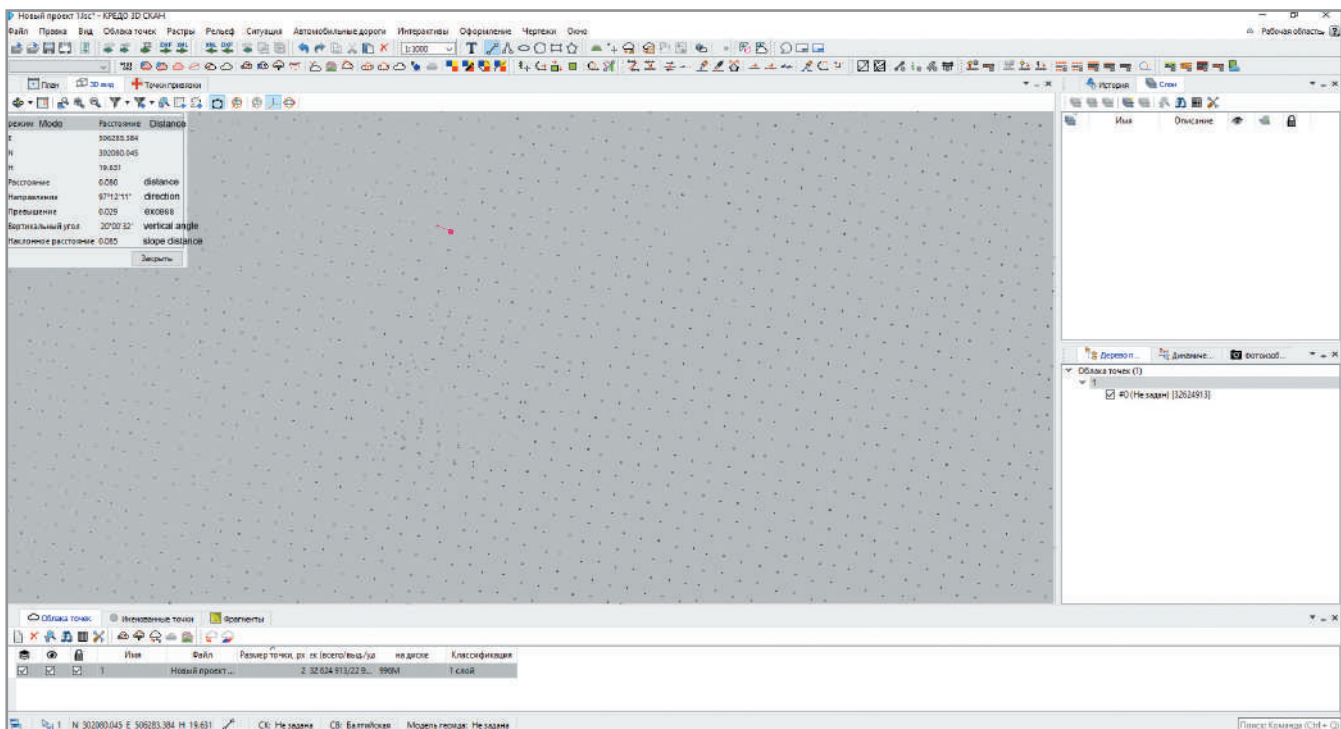


Fig. 1. CREDO 3D SCAN Program data on the distance between points

The distance is 8 cm. The actual area of the land plot can be calculated with high accuracy using photogrammetric data. This article will not describe the accuracy of photogrammetric data obtaining. Let's assume that this data meets the approved requirements¹.

mode	Distance
E	506283.384
N	302080.045
H	19.631
Distance	0.080
Direction	97°12'11"
Excess	0.029
Vertical angle	20°00'32"
Inclined distance	0.085

Fig. 2. Enlarged fragment of the CREDO 3D SCAN Program data

It can be predicted that in the near future the information on the entire surface of the territory of the Russian Federation will be collected and deciphered. In this context, universal and fairly “fast” algorithms for processing spatial information and calculating the actual areas of land plots will be sought after.

To speed up the actual area calculations, two new methods need to be developed and described: the “spatial triangular pyramid method” and the “spatial quadrangular pyramid method” – the “spatial envelope method”. These methods have scientific novelty, since in the analysis of many available methods, a similar methodology for calculating the physical surface area of land plots is not used.

2.1 Spatial triangular pyramid method

An arbitrary spatial triangle formed by three points with the obtained coordinates is the basis of a triangular pyramid in three-dimensional space $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$. Another point $D(x_4, y_4, z_4)$ lies outside the triangle plane, forming a pyramid ABCD, Figure 3.

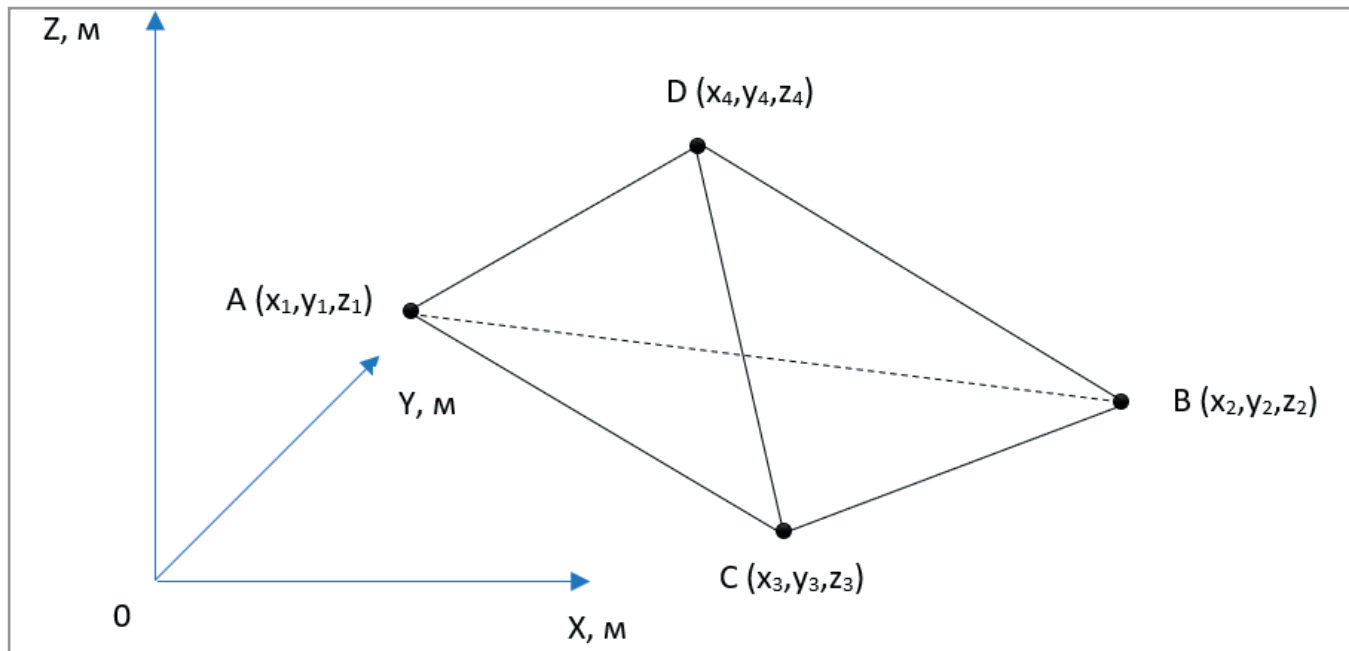


Рис. 3. Треугольная пирамида в трехмерном пространстве

¹ Order of the Federal Service for State Registration, Cadastre and Cartography dated October 23, 2020, No. P/ 0393 On approval of requirements for accuracy and methods for determining the coordinates of characteristic points of land plot boundaries, accuracy requirements and methods for determining the coordinates of characteristic points of the contour of a building, structure or construction in progress on a land plot, as well as requirements for determining the area of a building, structure, premises, and parking space”.

The lateral surface area of the pyramid ABCD will be the areas sum of the three triangles ACD, BCD, ABD.

$$S_{l.tr.pyr.} = S_{\Delta ACD} + S_{\Delta BCD} + S_{\Delta ABD} \quad (1)$$

So, you can calculate the areas of three spatial triangles at once, using the data of four points. The total actual land plot area is the sum of areas of all the side faces of the triangle pyramids in the coverage area of the land plot. This method allows a three-fold decrease in the time spent on calculating the actual area of a land plot, compared to calculating the actual area using three points, that is, by adding the areas of spatial triangles.

To find the areas of the lateral faces of a triangular prism, we apply the vector product of vectors and analytic geometry formulas:

$$S_{\Delta ACD} = \frac{\left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{AC} & \vec{AD} \end{matrix} \right|}{2} \quad (2)$$

$$S_{\Delta CBD} = \frac{\left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{CB} & \vec{CD} \end{matrix} \right|}{2} \quad (3)$$

$$S_{\Delta ABD} = \frac{\left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{AB} & \vec{AD} \end{matrix} \right|}{2} \quad (4)$$

Then, substituting formulas (2), (3), (4), in (1) we obtain:

$$S_{6.tr.pmp} = \frac{1}{2} \left(\left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{AC} & \vec{AD} \end{matrix} \right| + \left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{CB} & \vec{CD} \end{matrix} \right| + \left| \begin{matrix} \vec{x} & \vec{y} \\ \vec{AB} & \vec{AD} \end{matrix} \right| \right) \quad (5)$$

We find the vector product of vectors through determinants:

$$\begin{aligned} \vec{x}_{AC} \vec{y}_{AD} &= \begin{vmatrix} i & j & k \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = (y_3 - y_1)(z_4 - z_1)i + (z_3 - z_1)(x_4 - x_1)j + \\ &+ (x_3 - x_1)(y_4 - y_1)k - (x_4 - x_1)(y_3 - y_1)k - (x_3 - x_1)(z_4 - z_1)j - \\ &- (y_4 - y_1)(z_3 - z_1)i = ((y_3 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_3 - z_1))i + \\ &+ ((z_3 - z_1)(x_4 - x_1) - (x_3 - x_1)(z_4 - z_1))j + \\ &+ ((x_3 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_3 - y_1))k \end{aligned}$$

So, the vector product modulus is equal to:

$$\begin{aligned}
 \left| \begin{matrix} \vec{X} \\ \vec{AC} \end{matrix} \begin{matrix} \vec{AD} \end{matrix} \right| &= \sqrt{\left((y_3 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_3 - z_1) \right)^2 +} \\
 &\quad \left((z_3 - z_1)(x_4 - x_1) - (x_3 - x_1)(z_4 - z_1) \right)^2 + \\
 &\quad \left((x_3 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_3 - y_1) \right)^2}
 \end{aligned} \tag{6}$$

The same will be calculated for the vector product modules.

$$\begin{aligned}
 &\left| \begin{matrix} \vec{X} \\ \vec{CB} \end{matrix} \begin{matrix} \vec{CD} \end{matrix} \right| \mathbf{i} \left| \begin{matrix} \vec{X} \\ \vec{AB} \end{matrix} \begin{matrix} \vec{AD} \end{matrix} \right| : \\
 \begin{matrix} \vec{X} \\ \vec{CB} \end{matrix} \begin{matrix} \vec{CD} \end{matrix} &= \begin{vmatrix} i & j & k \\ x_2 - x_3 & y_2 - y_3 & z_2 - z_3 \\ x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \end{vmatrix} = (y_2 - y_3)(z_4 - z_3)i + (z_2 - z_3)(x_4 - x_3)j + \\
 &+ (x_2 - x_3)(y_4 - y_3)k - (x_4 - x_3)(y_2 - y_3)k - (x_2 - x_3)(z_4 - z_3)j - \\
 &- (y_4 - y_3)(z_2 - z_3)i = \left((y_2 - y_3)(z_4 - z_3) - (y_4 - y_3)(z_2 - z_3) \right) i + \\
 &\quad + \left((z_2 - z_3)(x_4 - x_3) - (x_2 - x_3)(z_4 - z_3) \right) j + \\
 &\quad + \left((x_2 - x_3)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_3) \right) k
 \end{aligned}$$

So, the vector product modulus is equal to:

$$\begin{aligned}
 \left| \begin{matrix} \vec{X} \\ \vec{CB} \end{matrix} \begin{matrix} \vec{CD} \end{matrix} \right| &= \sqrt{\left((y_2 - y_3)(z_4 - z_3) - (y_4 - y_3)(z_2 - z_3) \right)^2 +} \\
 &\quad \left((z_2 - z_3)(x_4 - x_3) - (x_2 - x_3)(z_4 - z_3) \right)^2 + \\
 &\quad \left((x_2 - x_3)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_3) \right)^2}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 \begin{matrix} \vec{X} \\ \vec{AB} \end{matrix} \begin{matrix} \vec{AD} \end{matrix} &= \begin{vmatrix} i & j & k \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = (y_2 - y_1)(z_4 - z_1)i + (z_2 - z_1)(x_4 - x_1)j + \\
 &+ (x_2 - x_1)(y_4 - y_1)k - (x_4 - x_1)(y_2 - y_1)k - (x_2 - x_1)(z_4 - z_1)j - \\
 &- (y_4 - y_1)(z_2 - z_1)i = \left((y_2 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_2 - z_1) \right) i + \\
 &\quad + \left((z_2 - z_1)(x_4 - x_1) - (x_2 - x_1)(z_4 - z_1) \right) j + \\
 &\quad + \left((x_2 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_2 - y_1) \right) k
 \end{aligned}$$

Then the vector product modulus will be equal to:

$$\begin{aligned}
 \left| \vec{x}_{AB} \times \vec{x}_{AD} \right| = & \sqrt{\left((y_2 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_2 - z_1) \right)^2 +} \\
 & \left((z_2 - z_1)(x_4 - x_1) - (x_2 - x_1)(z_4 - z_1) \right)^2 + \\
 & \left((x_2 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_2 - y_1) \right)^2}
 \end{aligned} \tag{8}$$

If we substitute the calculated values of the moduli of vector products obtained in formulas (6), (7), (8) into formula (5), we get formulas for calculating the lateral surface area of an arbitrary triangular pyramid in three-dimensional space, knowing the coordinates of the vertices of the points forming this pyramid:

$$\begin{aligned}
 S_{\text{б.тр.пир}} = & \frac{1}{2} \left(\sqrt{\left((y_3 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_3 - z_1) \right)^2 +} \right. \\
 & \left. \left((z_3 - z_1)(x_4 - x_1) - (x_3 - x_1)(z_4 - z_1) \right)^2 +} \right. \\
 & \left. \left((x_3 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_3 - y_1) \right)^2 +} \right. \\
 & \left. \sqrt{\left((y_2 - y_3)(z_4 - z_3) - (y_4 - y_3)(z_2 - z_3) \right)^2 +} \right. \\
 & \left. \left((z_2 - z_3)(x_4 - x_3) - (x_2 - x_3)(z_4 - z_3) \right)^2 +} \right. \\
 & \left. \left((x_2 - x_3)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_3) \right)^2 +} \right. \\
 & \left. \sqrt{\left((y_2 - y_1)(z_4 - z_1) - (y_4 - y_1)(z_2 - z_1) \right)^2 +} \right. \\
 & \left. \left((z_2 - z_1)(x_4 - x_1) - (x_2 - x_1)(z_4 - z_1) \right)^2 +} \right. \\
 & \left. \left((x_2 - x_1)(y_4 - y_1) - (x_4 - x_1)(y_2 - y_1) \right)^2 \right)
 \end{aligned} \tag{9}$$

Basing on the formula (9), it is possible to write a formula applicable for a general case and convenient for computer programming:

$$\begin{aligned}
 S_{\text{б.тр.пир } n} = & \frac{1}{2} \left(\sqrt{\left((y_{n+2} - y_n)(z_{n+3} - z_n) - (y_{n+3} - y_n)(z_{n+2} - z_n) \right)^2 + \right. \\
 & \left. + \left((z_{n+2} - z_n)(x_{n+3} - x_n) - (x_{n+2} - x_n)(z_{n+3} - z_n) \right)^2 + \right. \\
 & \left. + \left((x_{n+2} - x_n)(y_{n+3} - y_n) - (x_{n+3} - x_n)(y_{n+2} - y_n) \right)^2 + \right. \\
 & \left. \sqrt{\left((y_{n+1} - y_{n+2})(z_{n+3} - z_{n+2}) - (y_{n+3} - y_{n+2})(z_{n+1} - z_{n+2}) \right)^2 + \right. \\
 & \left. + \left((z_{n+1} - z_{n+2})(x_{n+3} - x_{n+2}) - (x_{n+1} - x_{n+2})(z_{n+3} - z_{n+2}) \right)^2 + \right. \\
 & \left. + \left((x_{n+1} - x_{n+2})(y_{n+3} - y_{n+2}) - (x_{n+3} - x_{n+2})(y_{n+1} - y_{n+2}) \right)^2 + \right. \\
 & \left. \sqrt{\left((y_{n+1} - y_n)(z_{n+3} - z_n) - (y_{n+3} - y_n)(z_{n+1} - z_n) \right)^2 + \right. \\
 & \left. + \left((z_{n+1} - z_n)(x_{n+3} - x_n) - (x_{n+1} - x_n)(z_{n+3} - z_n) \right)^2 + \right. \\
 & \left. + \left((x_{n+1} - x_n)(y_{n+3} - y_n) - (x_{n+3} - x_n)(y_{n+1} - y_n) \right)^2 \right),
 \end{aligned} \tag{10}$$

where n varies from 1 to m in steps of 2.

The actual area of the land plot is the sum of all the areas of the side faces of triangular pyramids:

$$S_{\text{зп}} = \sum_{n=1}^m S_{\text{б.тр.пир.}}$$

Using the obtained formulas (10) and (11), we can calculate the actual area of the land plot and record it in the three-dimensional real estate cadastre. The photogrammetric data depicts the surveyed topography rather accurately, as described above.

2.2 Spatial quadrangular pyramid method

An arbitrary spatial quadrangle formed by four points with the obtained coordinates is the basis of a quadrangular pyramid in three-dimensional space A (x₁, y₁, z₁), B (x₂, y₂, z₂), C (x₃, y₃, z₃), D (x₄, y₄, z₄). Another point E (x₅, y₅, z₅) lies outside two planes passing through points ABC and ADC, forming a conditional pyramid ABCDE, Figure 4.

Area calculations in this case are similar to the area calculations of the lateral surface of a triangular pyramid. You can calculate the areas of four spatial triangles at once, using the data of five points. The total actual land plot area is the sum of the areas of all the lateral faces of the

quadrangular pyramids in the coverage area of the land plot. This method allows a four-fold decrease in the time spent on calculating the actual area of a land plot, compared to calculating the actual area using three points, that is, adding the areas of triangles.

The further enlargement of the used points is unreasonable due to the calculation awkwardness and increase in the time spent on calculations and summing up all the data obtained.

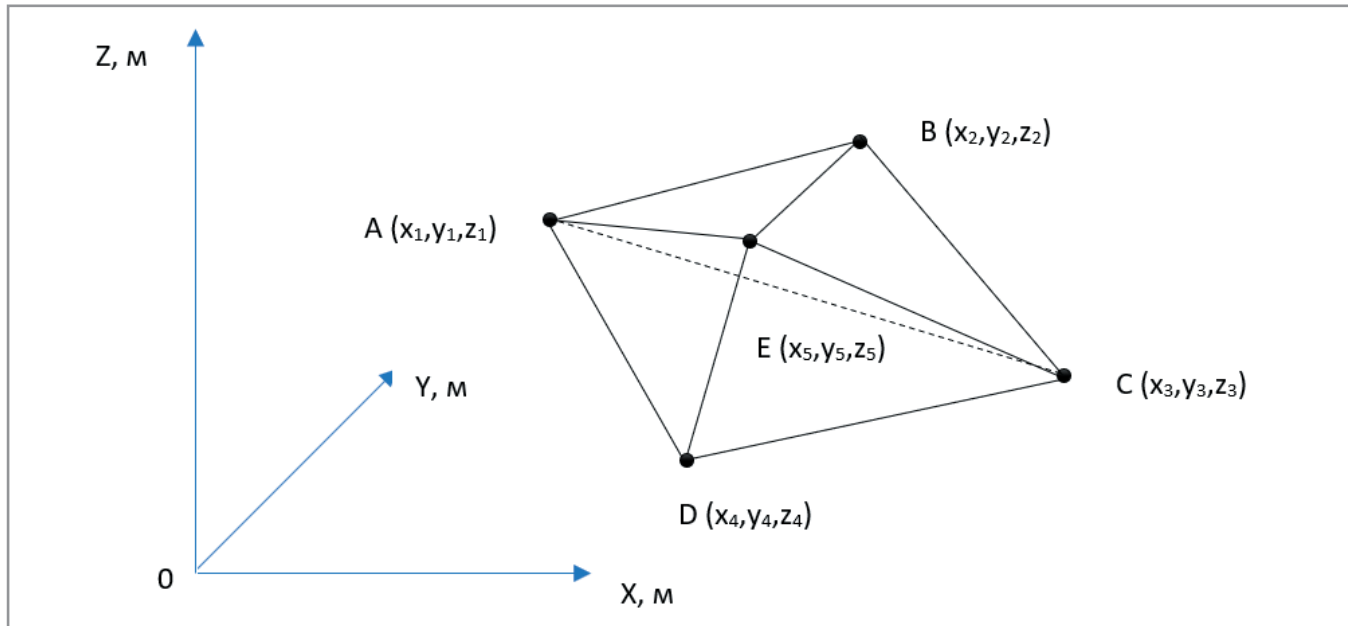


Fig. 4. Quadrangular pyramid in three-dimensional space

3 Conclusion

Shifting from a conventional two-dimensional cadastre to a three-dimensional real estate cadastre, the data both on the land plot area projection onto the plane and on the actual area of the land plot should be entered into the Unified State Register of Real Estate.

There are two methods developed and described – “the method of the spatial triangular pyramid” and the “method of the spatial quadrangular pyramid”. These methods allow the calculation of the actual area of the land plot with sufficient accuracy and in less time. Also, this allows a three- or four-fold reduction of the calculations of the actual area of land plots, respectively.

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